Although the mean period of RR Lyrae is remarkably constant, conspicuous transitory irregularities are present both in the period and in the form of the light curve, much resembling those of the cluster-type variable XX Cygni previously discussed in these PROCEEDINGS. In particular, there is a slow but definite variation in the length of the period which seems to complete its cycle in about sixteen years; and an oscillation in the time of the rise to maximum light which at times appears to have a uniform period of about forty days, and at other times to be erratic in both period and amplitude. The formula:

$$\text{Max.} = 2414856.451 + 0\overset{d}{.}566831\text{E} - 0\overset{d}{.}024 \sin (0\overset{\circ}{.}0340\text{E} - 104\overset{\circ}{.}5)$$
$$+ [0\overset{d}{.}004 + 0\overset{d}{.}013 \cos \frac{2\pi}{70} (\text{E} - 45)]$$

in which E represents the number of periods, predicts the individual maxima with high accuracy for the first seven of eight hundred periods after the initial epoch, and by dropping the short period harmonic it gives the mean maxima accurately for at least fifteen years from the initial date. The complete discussion of these features, based on all the available series of light measures and on the recent observations of the spectrum, will be published as a *Contribution from the Mount Wilson Solar Observatory*.

THE SPECTRUM OF δ CEPHEI

By Walter S. Adams and Harlow Shapley

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

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About 130 years ago Goodricke discovered the periodic fluctuations in the light of the naked-eye star δ Cephei—the eighth variable known to astronomers. None of those previously recorded exhibited the type of light variation peculiar to this fourth magnitude star, but of the four or five thousand variables found since that time more than half show the same character of light fluctuations. The class has, accordingly, received the appropriate designation of Cepheid variables. A century after the discovery of the light variations, δ Cephei was also found to vary in radial velocity—the sixth spectroscopic variable to be placed on record. Again the variation differed from that of other stars. The period of the oscillation of the spectral lines is identical with that of the light variation, but the orbital elements, derived on the hypothesis that the star is a binary, permit no satisfactory explanation of the light variations. When other velocity-variables of this character were found

they were classed as Cepheids, and, in fact, every light-variable of the Cepheid type, when put to the test, has proved to be a velocity-variable with the distinct Cepheid peculiarities. Any definite contribution to the explanation of the light or velocity variations of a single Cepheid, therefore, is of unusual value in that it involves the interpretation of the majority of all variable stars.

The present state of our knowledge of the causes of Cepheid variation has been discussed in former papers. The clearest outcome of recent studies of the subject is the apparent impossibility of relating the velocity variations to orbital motions. Probably the strongest evidence on this point is the synchronous variations of the magnitude and the spectral class, recently observed at Mount Wilson for two cluster-type Cepheids. Such variations of spectrum have received little consideration heretofore, notwithstanding the known change in color from maximum to minimum light that would seemingly demand the changes in spectral lines that distinguish one class of spectrum from another.

The question naturally arises whether in suitable investigations of the brightest and most typical Cepheids changes of spectrum would not be found analogous to those observed in the less typical variables RS Boötis and RR Lyrae. Might it not also be possible with a special analysis of high dispersion plates to bring out other characteristics of the spectrum that would throw light on the causes underlying the light and velocity variations? With these points in view an investigation of the spectrum of δ Cephei was made, and the results are briefly outlined in the following paragraphs. The discovery of the conspicuous change of the spectral class of the first and best known Cepheid indicates that constancy of spectrum is to be expected for none of this class of variables, and, coupled with the remarkable behavior of the spectral lines, suggests that the possibility of a completely satisfactory theory of Cepheid variation is not necessarily remote.

Two photographs of the spectrum of δ Cephei were obtained on the nights of December 23 and 24, 1915, with the Cassegrain spectrograph and 60-inch reflector. The full optical train of three prisms and a 102 cm. camera was employed. This combination gives a linear scale at $H\gamma$ of 5.3 angstroms to the millimeter. The data for the photographs are as follows:

Plate γ 4571	Dec. 23, G.M.T. 15^h4^m	Exposure time 225^m
Plate γ 4578	Dec. 24, G.M.T. 14^h7^m	Exposure time 120^m

The first photograph was taken under exceptionally poor conditions, and this fact is responsible for the extended exposure time. As it is,

the plate, though well measurable, is somewhat under-exposed. Especial care was taken in adjusting the spectrograph to secure good definition in the region of the hydrogen line $H\gamma$. As a result the spectrum lines are very sharply defined from λ 4200 to λ 4500, but begin to show diffuseness to the red of this point. They are, however, measurable as far as λ 4600.

A comparison of the two negatives under a Hartmann spectrocomparator at once showed some important differences beween the two spectra.[1] These may be summarized as follows. The photograph of

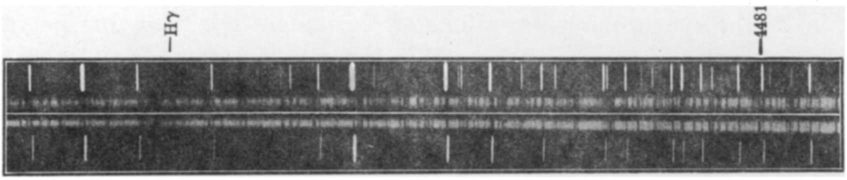

FIG. 1. SPECTRA OF δ CEPHEI
Top: Spectrum near Minimum of Light
Bottom: Spectrum near Maximum of Light

December 24 was taken near the star's maximum of light, and that of December 23 not far from minimum.

	γ 4578 (*maximum*)	γ 4571 (*minimum*)
Hydrogen line $H\gamma$	Strong	Much weakened
Enchanced lines of *Fe*, *Ti*, *Sr*, and *Cr*	Strong	Much weakened
λ 4481, enhanced *Mg*	Very strong	Much weakened
λ 4227 of calcium	Strong	Strengthened
Low temperature lines of *Ca*, *Fe*, *Ti*, and *Cr*	Weak	Strengthened
Continuous spectrum	Strong in violet	Weakened in violet

The observation on the continuous spectrum is made somewhat uncertain by the general under-exposure of the plate taken near minimum, but the result is apparently as given. The lines on this photograph appear to be broader and less sharply defined than those on the plate at maximum, and this effect we consider as probably genuine after all necessary allowance has been made for the difference in the quality of the two negatives. There is, however, no evidence of lack of symmetry in the spectrum lines either at maximum or minimum, nor of the presence of a secondary spectrum. It appears from these observations that the changes follow a definite tendency. At maximum the high temperature lines are very strong, and the low temperature lines are weak; while at minimum the reverse is the case. The conclusion appears to be justified that the temperature of the gases constituting the star's absorbing envelope is higher at maximum of light than at minimum.

A method of applying the Harvard system of classification to the determination of spectral type through the aid of numerical relationships between the intensities of the hydrogen lines and certain other selected lines will be described by one of us in another Communication. The use of this method gives in the case of δ Cephei:

At maximum F4, At minimum G2

The variation in type, accordingly, amounts to 8 divisions of the Harvard scale.

An independent determination of the spectrum and its change can be made from the comparison of the mean visual and photographic light curves. Nearly twenty visual curves have been made during the last hundred years, but only the one by Stebbins is based on measures with a photometer. The only photographic curve is due to Wirtz. It is possible to reduce both curves to the international magnitude scale; the difference between them for any phase gives the color index, which may be transformed into spectral type by known relations. Hence we derive in stellar magnitudes:

$$\text{Visual Range} = 4.25 - 3.49 = 0.76,$$
$$\text{Photographic Range} = 5.15 - 3.90 = 1.25;$$

and for the spectral class at light maximum F2, at minimum G4. The time of plate γ 4571 coincides with velocity minimum but succeeds the minima of the light and color curves by several hours. Allowing for this we derive from the color curve, for comparison with the direct classification above, the satisfactory result:

At maximum F2, At minimum G0

An important conclusion is that all genuine color variations observed in Cepheids may be directly interpreted as normal changes in spectral class.

Associated with the variation in spectral type in δ Cephei is a variation in the intensity of certain spectrum lines which have been found to fluctuate with the intrinsic luminosities of the stars in which they occur. The use of the intensities of these lines in calculations of absolute stellar magnitudes will be described elsewhere. The application of the method to the case of δ Cephei is of considerable interest because of the accurately known range of variation in apparent, and hence, of course, in absolute magnitude. Unfortunately the spectrum of the star at maximum (F4) is of a type not well suited for the use of the method and the results are necessarily approximate. The variation in absolute magnitude found in this way, however, is entitled to considerably more weight than are

the magnitude values themselves. The two photographs give the following values:

Minimum M = +1.8, Maximum M = +0.7, Variation 1.1

The two conclusions to be drawn from these results are: First, that they agree with results based on other considerations such as proper motion in indicating that δ Cephei is a star of very high intrinsic brightness; second, the variation in magnitude derived from characteristics of the spectrum, 1.1, is in very fair accord with the value 0.8, the star's known range of visual magnitude.

In selecting the lines to be measured for the determination of radial velocity from the two photographs the consideration was borne in mind that different lines might give different velocities. Accordingly, an extended list was made out including: (1) a large number of enhanced lines; (2) iron lines which show a wide variety of displacements under pressure; (3) special lines such as $H\gamma$ and λ 4227 of calcium. The photograph taken near maximum was measured by Mrs. Monk and Adams, the result given being the mean for the two observers; the other plate was measured by Mrs. Monk alone:

	No. of Lines	*Radial Velocity*
γ 4571 (minimum)	55	+ 3.8 km/sec
γ 4578 (maximum)	68	−35.2

The radial velocity for the times of the Mount Wilson photographs can be predicted by means of the elements of the velocity variation, derived by Moore[2] from Lick Observatory spectrograms, using in this computation the improved light elements by Luizet.[3] The results are:

Minimum, +3.7 km/sec, Maximum, −35.2.

The exact agreement between the observed and computed velocities not only checks the results from the two observatories and indicates that the range of velocity variation is the same now as eight years ago, but also supports Moore's conclusion that the velocity of the center of mass does not undergo the variation assigned it by Belopolsky.

A comparison of the velocities given by the individual lines on these photographs shows some interesting features. Of these the most important is the behavior of the iron lines which in the laboratory show small displacements under pressure as compared with those which show relatively large shifts. The results for the photograph taken near the star's maximum are shown in detail in Table I. The pressure shifts in angstrom units per atmosphere, given under Δ, are taken from unpublished laboratory results by Gale and Adams. The velocities, v, are in kilometers, and are the measured values, not corrected for the earth's motion.

The mean value of the velocity as derived from the lines of large pressure shift is 1.58 km greater than that from those of small shift, and it is noteworthy that not one of the velocities of the lines in the latter group reaches the mean value for the large shift lines. Similarly only two of these reach values as low as the mean for the lines of small pressure displacement.

The photograph taken near the star's minimum shows similar results, although it is entitled to considerably less weight because of its quality and the fact that but a single measurement has been made upon it.

TABLE I

Small pressure shift

LINE	Δ A	V KM	LINE	Δ A	V KM
4250.9	+0.001	−20.1	4415.2	+0.004	−20.4
4271.9	0.001	18.9	4422.7	0.001	21.2
4325.9	0.002	21.2	4466.7	0.003	19.5
4352.9	0.002	19.6	4442.5	0.005	19.6
4376.1	0.002	18.3	4482.3	0.003	20.1
4383.7	0.003	20.0	4531.3	0.003	18.3
4404.9	0.004	19.9			
Mean........				+0.0027	−19.79

Large pressure shift

LINE	Δ A	V KM	LINE	Δ A	V KM
4222.3	+0.007	−20.8	4271.3	+0.008	−22.6
4227.6	0.017	24.0	4407.8	0.012	20.6
4233.7	0.006	20.3	4430.7	0.007	19.6
4236.1	0.007	23.7	4447.8	0.006	19.4
4238.9	0.012	22.5	4494.7	0.007	20.5
4250.2	0.011	21.5	4528.7	0.006	21.7
4260.6	0.010	20.5			
Mean........				+0.0090	−21.37

Combining the results with weights of two and one we obtain the following value for the difference in velocity:

Large Δ lines (+0.009 A) − Small Δ lines (+0.003 A) = −1.22 km

The sign of this difference is opposite to that which would be found if pressure is the agent producing it and if the effective pressure at the star's surface where these lines are formed is greater than one atmosphere. It is, however, the sign which would be found if the stellar pressure is less than one atmosphere. In that case the iron comparison lines on the negatives, which of course are taken at atmospheric pressure,

would be shifted toward longer wave-lengths with reference to the stellar lines by amounts corresponding to the observed pressure displacements. It seems to us probable that this is the explanation of at least the principal part of the observed difference, although differential radial currents in the star's atmosphere may account for a portion of it.

An important confirmation of this result is furnished by Mr. St. John's measurements of the displacements of solar lines relative to the arc spectrum lines of iron. He has kindly furnished us with the provisional values obtained for the lines given in Table I, and from these we find: first, that the small pressure shift lines are displaced toward the red in the sun relative to the arc lines by an average amount of 0.009 angstrom; second, that the large pressure shift lines are displaced toward the violet 0.004 angstrom. These differences are in the same direction as in the case of δ Cephei and the absolute amount is of much the same order. Thus we find:

Sun	Large Δ lines	$-$	Small Δ lines	$=$	-0.013 A	$=$	-0.9 km
δ Cephei	"		"			$=$	-1.2 km

It is well known that the pressures in the upper portions of the sun's atmosphere where the stronger lines of the spectrum appear to find their origin are extremely low.

A comparison of the velocities given by the enhanced lines in δ Cephei with those given by such as are not enhanced indicates a distinct shift of the former toward longer wave-lengths. In the case of the photograph taken near maximum the difference amounts to $+0.86$ km, or about $+0.012$ angstrom. The other photograph gives a similar value of larger amount. A difference of the same character was found by Adams some years ago for the enhanced lines in the spectra of Sirius and Procyon.[4] The values were $+0.90$ km for Sirius and $+0.58$ for Procyon, with which the result found for δ Cephei is in very fair agreement. It is probable that the explanation of these results is to be found mainly in radial convection currents in the stellar atmospheres.

[1] Belopolsky and Lohmann at Pulkova have described some differences in the intensities of the lines at maximum and minimum light, but the dispersion employed makes it impossible to identify the lines with certainty.

[2] Moore, *Lick Obs. Bull.*, 7, 153 (1913).

[3] Luizet, *Ann. l'Univ. Lyon*, N. S., 33, 42 (1912).

[4] *Astrophys. J.*, 33, (1911).